

# **The Interaction of Internal and External Information in a Problem-Solving Task**

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# The Interaction of Internal and External Information in a Problem-Solving Task

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JIAJIE ZHANG

## ABSTRACT

*In these studies I examine the role of distributed cognition in problem solving. The major hypothesis explored is that intelligent behavior results from the interaction of internal cognition, external objects, and other people, where a distributed cognitive task can be represented as a set of modules, some internal and some external. The Tower of Hanoi problem is used as a concrete example for these studies. In Experiment 1 I examine the effects of the distribution of representations among internal and external modules on problem-solving behavior. Experiments 2A and 2B focus on how the nature and number of rules affect problem-solving behavior. Experiment 3 investigates how a group's problem-solving behavior is affected by the distribution of representations among the individuals. The results of all studies show that distributed cognitive activities are produced by the interaction among the representations in the modules involved in a given task: between internal and external representations and between internal representations. External representations are not peripheral aids. They are an indispensable part of cognition. Two of the factors determining the performance of a distributed cognitive system are the structure of the abstract task space and the distribution of representations across modules.*

## INTRODUCTION

The traditional approach to cognition in general and problem solving in particular focuses on an individual's internal mental states. In the traditional view, representation and cognition are exclusively the activity of an internal mind. External objects, if they have anything to do with cognition at all, are at most peripheral aids. The cognitive properties of a group are solely determined by the structures internal to the individuals. There is no doubt that internal factors are important to cognition. They are not, however, the whole story. Much of a person's intelligent behavior results from interactions with external objects and with other people. External and social factors also play critical roles in cognitive activities. Recently, cognitive scientists have started to address "distributed cognition," the study of how cognitive activity is distributed across internal human minds, external cognitive artifacts, groups of people, and across space and time (Hutchins, 1990, in preparation; Hutchins & Norman, 1988; Norman, 1988, 1989, 1990). In the study of cognitive artifacts, Norman (1990) argues that artifacts not only enhance a person's ability to perform a task, but they

also change the nature of the task. In the study of the social organization of distributed cognition, Hutchins (1990) shows that social organizational factors often produce group properties that differ considerably from the properties of individuals.

In this paper, I develop a framework, the modularity of representations, to analyze a set of distributed cognitive tasks, and to explore the interactions among internal and external representations and among members of a group of people. I show that external objects are not simply peripheral aids—they provide a different form of representation. External representations are interwoven with internal representations to produce distributed cognitive activities. In addition, the share of knowledge among a set of modules is important for a system's performance.

### **Modularity of Representations**

The basic principle to be explored is that the representational system for a given task can be considered as a set, with some members internal and some external. Internal representations are in the mind, as propositions, mental images, or whatever (e.g., multiplication tables, arithmetic rules, logic, language, etc.). External representations are in the world, as physical symbols (e.g., written symbols, beads of an abacus, etc.) or as external rules, constraints, or relations embedded in physical configurations (e.g., spatial relations of the items in a table, spatial configurations of the digits on a piece of paper, physical constraints in an abacus, etc.). The representations discussed in this paper are representations for tasks. In this sense, we can speak of not only internal representations, which have their traditional meaning, but also external representations. For example, an external representation can represent the external part of the structure of a task. Generally, there are one or more internal and external representations involved in any task. Each representation is a

relatively isolated functional unit in a specific medium. I call this unit, whether internal or external, a module.

Figure 1 shows a representational system for a task with two internal and two external modules. Each internal module resides in a person's mind and each external module resides in an external medium. The representations of internal and external modules involved in a given task together form a distributed representation space mapped to a single abstract task space that represents the properties of the problem. Each module sets some constraints on the abstract task space. The distributed representation space plays an important role in the studies reported here.

The distributed cognition perspective demands the decomposition of the abstract task space into its internal and external components, because many cognitive tasks are distributed among internal and external modules. In the traditional studies of problem solving, many abstract task spaces having internal and external components were mistakenly treated as solely internal task spaces. Generally speaking, the abstract task space of a task is not equivalent to its internal task space.

### **The Tower of Hanoi**

The Tower of Hanoi problem<sup>1</sup> (Figure 2A) was chosen as a concrete example to study distributed cognitive activities in problem solving. The task of the Tower of Hanoi problem is to move all the disks from the left pole to the right pole, following two rules:

Rule 1: Only one disk can be moved at a time.

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<sup>1</sup> The disk sizes of the standard version of Tower of Hanoi are the reverse of those shown in Figure 2a: the largest disk is at the bottom and the smallest is at the top. The disk sizes have been reversed to make the experimental designs of all conditions consistent.

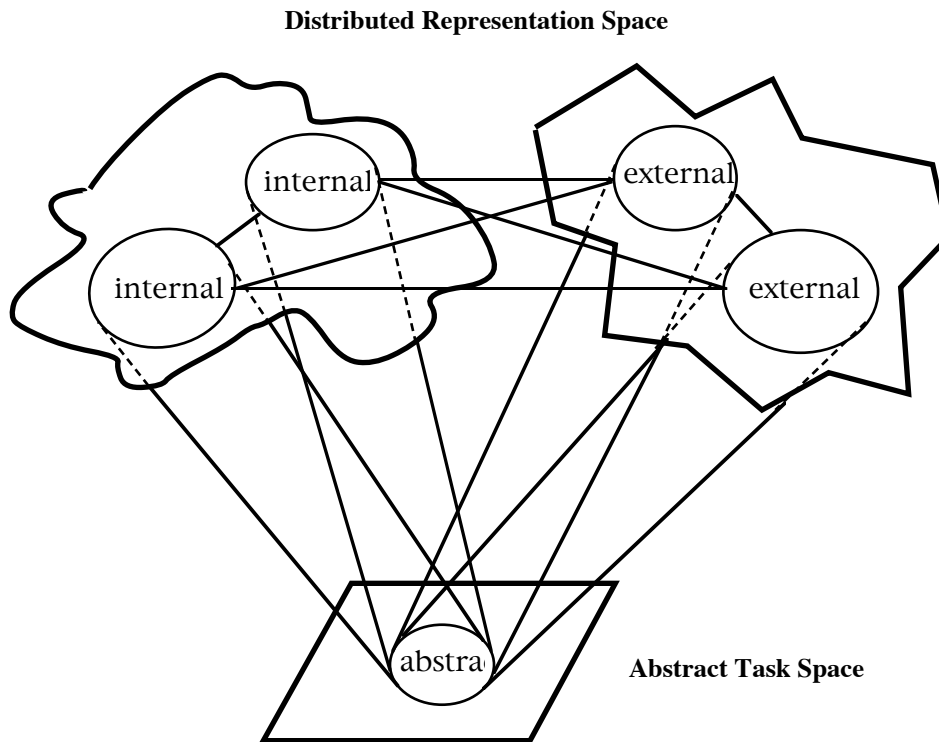


FIGURE 1. The distributed representation space and the abstract task space of a task with two internal and two external modules. The abstract task space is formed by a combination of internal and external representations.

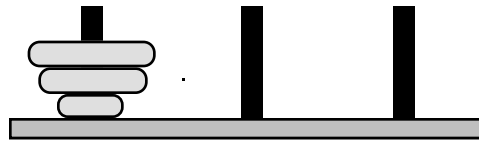
Rule 2: A disk can only be moved to another pole on which it will be the largest.

The problem space for the Tower of Hanoi (Figure 2B) shows all possible states and legal moves. Each rectangle shows one of the 27 possible configurations of the three disks on the three poles. The lines between the rectangles show the transformations from one state to another when the rules are followed.

The Tower of Hanoi is a well-studied problem (Hayes & Simon, 1977; Kotovsky & Fallside, 1988; Kotovsky, Hayes, & Simon, 1985; Simon & Hayes, 1976). Much of the research has focused on isomorphs of the Tower of Hanoi and their problem representations. The basic finding is that different problem representations can have dramatic impact on problem difficulty even if the formal structures are the same. External

memory aid is one major factor of problem difficulty. Thus, Kotovsky et al. (1985) reported that the Dish-move isomorph of the Tower of Hanoi, in which all rules had to be remembered, was harder to solve than the Peg-move isomorph, in which one of the rules was embedded in physical configurations. Modifications of these two isomorphs were used in two of the three conditions in Experiment 1 of the present study (Waitress and Orange and Waitress and Donuts).

*Internal and External Rules.* The Tower of Hanoi problem actually has three rules, not just the two stated earlier. Rule 3 is that only the largest disk on a pole can be transferred to another pole. In the representation shown in Figure 2A, Rule 3 need not be stated explicitly because the physical structure of the disks and poles coupled with Rules 1 and 2 guarantee that it will be followed. But if the



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A)

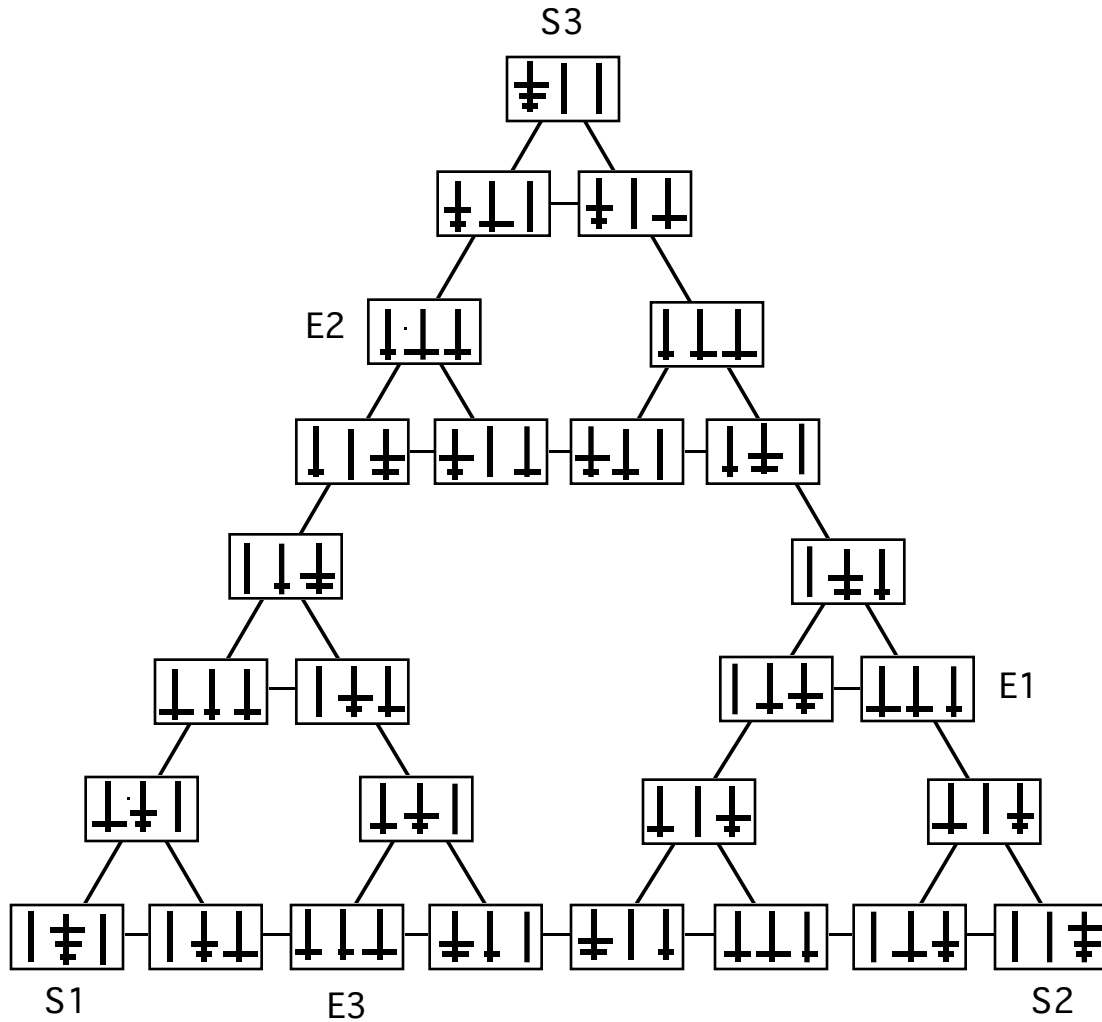


FIGURE 2. The Tower of Hanoi problem. (a) The task is to move all three disks from the left pole to the right pole. (b) The problem space of the Tower of Hanoi problem. Each rectangle shows one of the 27 possible configurations (states) of the three disks on the three poles. The lines between the rectangles show the transformations from one state to another when the rules are followed. S1, S2, and S3 are three starting states, and E1, E2, and E3 are three ending states. They will be used later.

disks were not stacked on poles, explicit statement of Rule 3 would be necessary.

In my studies I used four rules:<sup>2</sup>

<sup>2</sup> The problems in Experiments 1 and 3 were isomorphs of the standard Tower of Hanoi problem which only has

Rule 1: Only one disk can be transferred at a time.

Rule 2: A disk can only be transferred to a pole on which it will be the largest.

Rule 3: Only the largest disk on a pole can be transferred to another pole.

Rule 4: The smallest disk and the largest disk can not be placed on a single pole unless the medium-sized disk is also on that pole.

Any of these four rules can be either internal (memorized) or external (externalized into physical constraints).<sup>3</sup> In the experiments that follow, I varied the numbers of external rules. In one condition, called Waitress and Oranges (Figure 5A), no rule is external. In a second condition, called Waitress and Donuts (Figure 5B), Rule 3 is external. In the Waitress and Coffee condition (Figure 5C), both Rules 2 and 3 are external. In the Waitress and Tea condition (Figure 7D), Rules 2, 3, and 4 are all external.

*Internal and External Problem Spaces.* A problem space is composed of all possible states and all moves constrained by the rules. Figures 3A-F show the problem spaces constrained by Rules 1, 1+2, 1+3, 1+2+3, and 1+2+3+4, respectively.<sup>4</sup> Lines with arrows are unidirectional. Lines without arrows are

bidirectional. Note that the arrow lines in Figure 3B are in exactly opposite directions of those in Figure 3C. This implies that Rules 2 and 3 are complementary. One important point is that these five spaces can represent internal problem spaces, external problem spaces, or mixed problem spaces, depending upon how the rules constructing them are distributed across internal and external modules. A problem space constructed by external rules is an external problem space, one constructed by internal rules is an internal problem space, one constructed by a mixture of internal and external rules is a mixed problem space. Figure 3B is the internal problem space of the standard version of the Tower of Hanoi because Rules 1 and 2 are internal. If the physical constraints imposed by the disks themselves are such that only one can be moved at a time (i.e., the disks are large or heavy), then Figure 3C is its external problem space because under this circumstance Rules 1 and 3 are both external. These two spaces form the distributed representation space of the Tower of Hanoi (Figure 4). The conjunction of these two spaces forms the abstract task space.

## Outline of the Experiments

Normally, a cognitive task can be distributed among a set of internal and external modules. Experiment 1 examines the effects of the distribution of representations among internal and external modules on problem-solving behavior. By an analysis of the problem spaces in Figure 3 we can see that the problem space's structure changes with the number of rules. This structural

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Rules 1, 2, and 3. Experiments 2A and 2B used all four rules.

<sup>3</sup> The rules written on the instruction sheets are not considered as external rules in the present study. These rules are internal in the sense that they are memorized by subjects before the games are played. Only those rules which are built into physical constraints and not told to the subjects are considered to be external.

<sup>4</sup> The four rules for the Tower of Hanoi are not fully orthogonal. Rules 2, 3, and 4 are orthogonal to one another, but Rules 2 and 3 are not orthogonal to Rule 1, because Rule 1 is the prerequisite of Rules 2 and 3 which are the restrictions on moving *one* object. When a problem solver's task has rules which rely on other rules, its problem space can only be drawn in the context of the rules in which its rules rely.

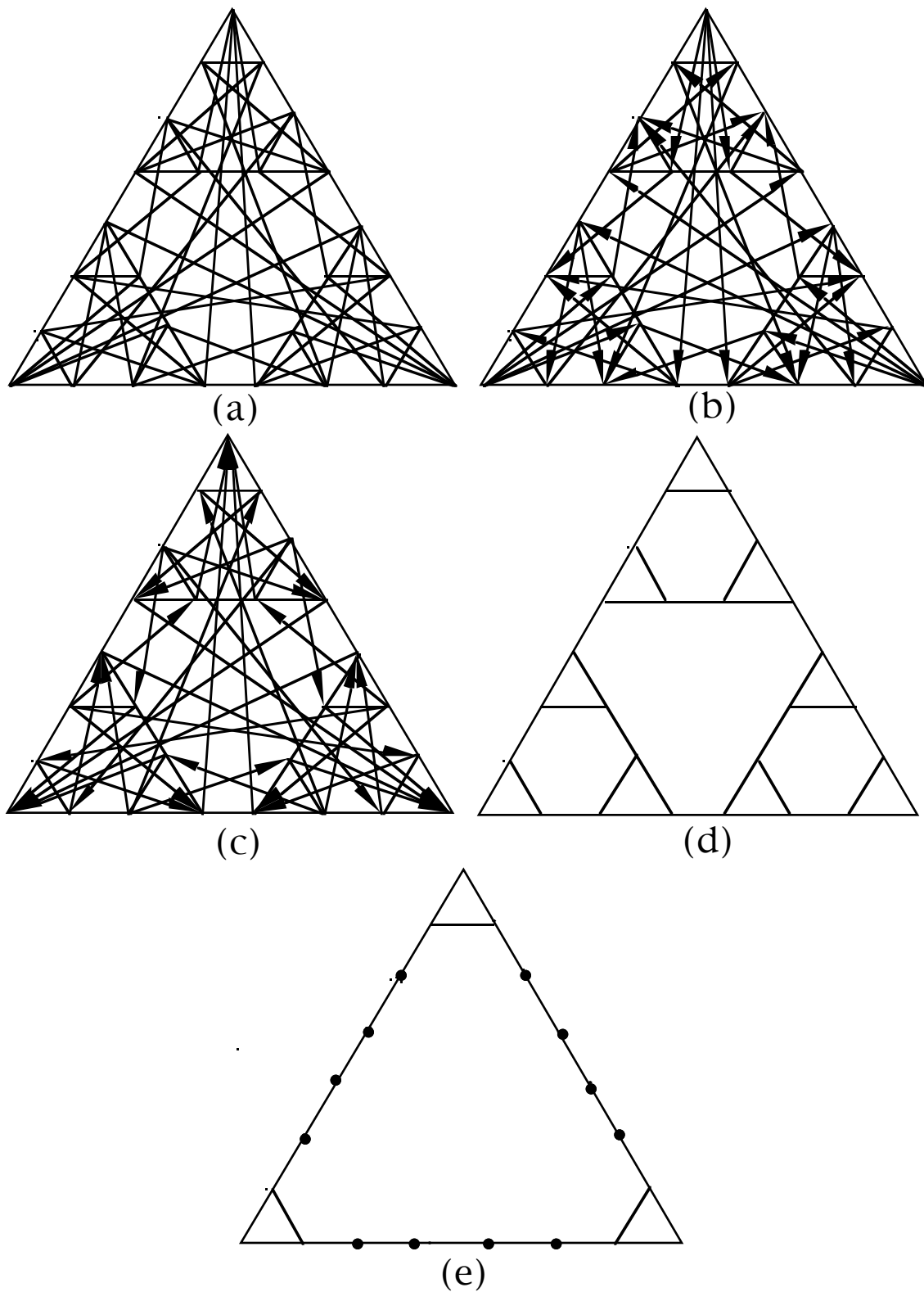


FIGURE 3. Problem spaces constrained by five sets of rules. (a) Rule 1. (b) Rules 1+2. (c) Rules 1+3. (d) Rules 1+2+3. (e) Rules 1+2+3+4. They are derived from the problem space in Figure 2b. Lines with arrows are uni-directional. Lines without arrows are bi-directional. The rectangles (problem states) are not shown in this figure for the reason of clarity. (a)-(d) have the same 27 problem states as in Figure 2b. (e) only has 21 problem states, which are the outer 21 rectangles in Figure 2b.

change might have an impact on the problem difficulty and, consequently, on problem-solving behavior. In addition, this impact, if any, might depend on the nature of the rules (internal or external). In Experiments 2A and 2B, the focus is on how the nature and number of rules affects problem solving behavior. A cognitive task can not only be distributed among internal and external modules, it can also be distributed among a set of internal modules. In Experiment 3, I investigate how a group's problem-solving behavior is affected by the distribution of representations among the individuals.

### EXPERIMENT 1

The standard Tower of Hanoi problem has three rules that can be distributed among internal and external modules. Different distributions may have different effects on problem-solving behavior, even if the formal structures are the same. Experiment 1 investigates these effects. My hypothesis is that the more rules are distributed in external modules, the better the system's performance. In addition, external rules might have some properties distinct from internal rules and hence change the behavior of a problem solver. There are three conditions, isomorphs

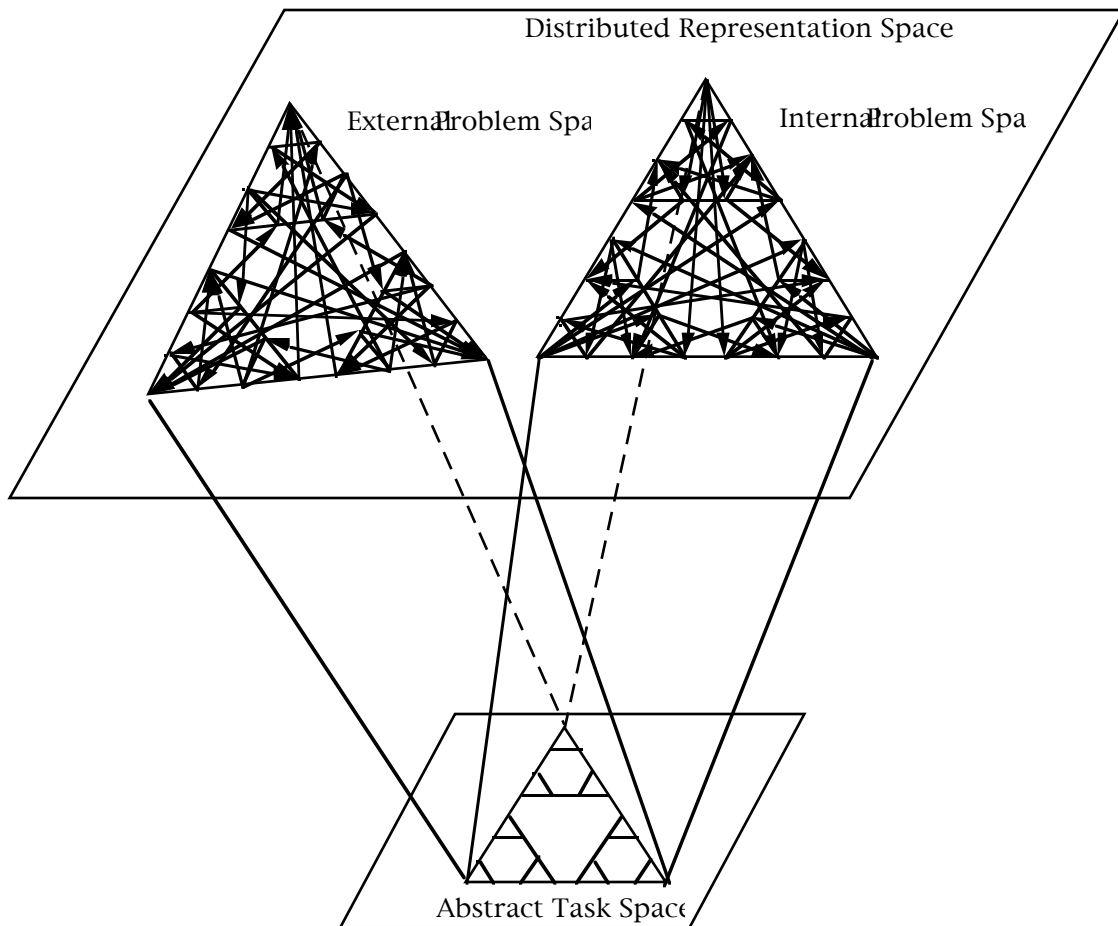


FIGURE 4. The distributed representation space and the abstract task space for the standard version of the Tower of Hanoi problem. The distributed representation space is composed of the internal and the external problem spaces, which are constrained by Rules 1+2 and Rules 1+3 (Rule 1 can be made external if the disks are big enough so that only one can be lifted at a time), respectively. The abstract task space is the conjunction of the internal and the external problem space.

of the Tower of Hanoi, which correspond to three different distributions of the three rules between an internal and an external module. I made up three restaurant stories to explain the three conditions (see Figure 5).

In the Waitress and Oranges (I123) condition, Rules 1, 2, and 3 were all internal (Figure 5A).

In the Waitress and Donuts (I12-E3) condition, Rules 1 and 2 were internal, and

Rule 3 was external. The physical constraints (coupled with Rules 1 and 2) guarantee that Rule 3 is followed. The verbal instructions for I12-E3 were the same as for I123, except that the word *orange* was replaced by the word *donut* and Rule 3 didn't appear in the instructions. (The pictorial part of the instructions is shown in Figure 5B.)

In the Waitress and Coffee (I1-E23) condition, Rule 1 was internal, and Rules 2 and 3

### Waitress and Oranges

A strange, exotic restaurant requires everything to be done in a special manner. Here is an example. Three customers sitting at the counter each ordered an orange. The customer on the left ordered a large orange. The customer in the middle ordered a medium sized orange. And the customer on the right ordered a small orange. The waitress brought all three oranges in one plate and placed them all in front of the middle customer (as shown in Picture 1). Because of the exotic style of this restaurant, the waitress had to move the oranges to the proper customers following a strange ritual. No orange was allowed to touch the surface of the table. The waitress had to use only one hand to rearrange these three oranges so that each orange would be placed in the correct plate (as shown in Picture 2), following these rules:

- Only one orange can be transferred at a time. (Rule 1)
- An orange can only be transferred to a plate in which it will be the largest. (Rule 2)
- Only the largest orange in a plate can be transferred to another plate. (Rule 3)

How would the waitress do this? That is, you solve the problem and show the movements of oranges the waitress has to do to go from the arrangement shown in Picture 1 to the arrangement shown in Picture 2.

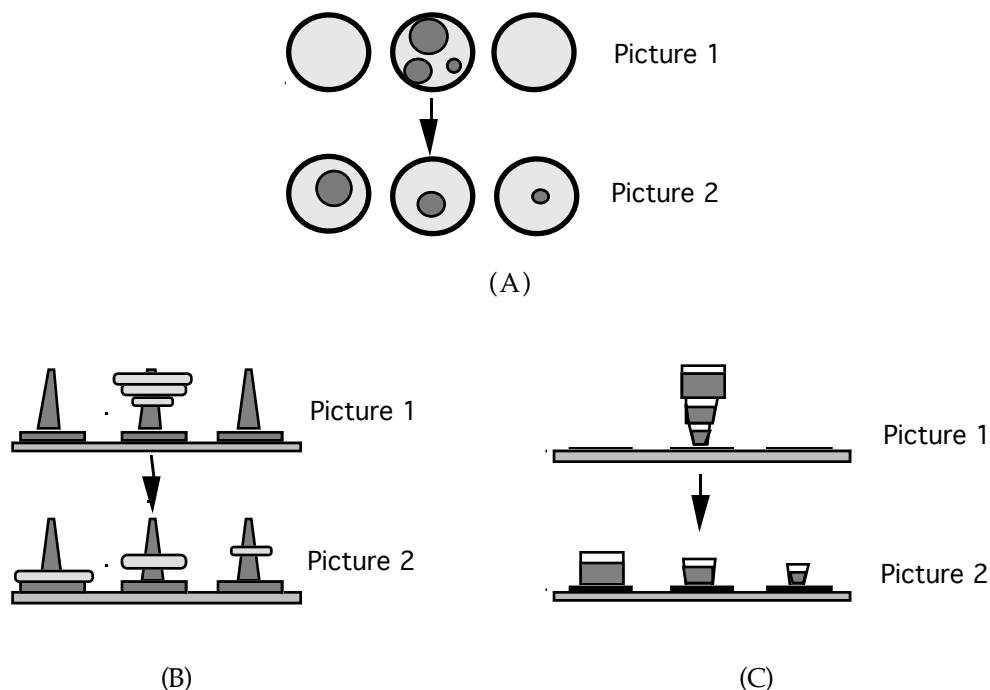


FIGURE 5. (a) The complete instruction for the *Waitress and Oranges* (I123) condition. (b) The pictorial part of the instructions for the *Waitress and Donuts* (I12-E3) condition. (c) The pictorial part of the instructions for the *Waitress and Coffee* (I1-E23) condition.

were external. A smaller cup could not be placed on the top of a larger cup (Rule 2), as this would cause the coffee to spill. A cup could not be moved if there was another cup on top of it (Rule 3). The verbal part of the instructions for I1-E23 was the same as for I123, except that the word *orange* was replaced by the words *cup of coffee* and Rules 2 and 3 didn't appear in the instructions. (The pictorial part is shown in Figure 5C).

## Method

*Subjects.* The subjects were 18 undergraduate students enrolled in introductory psychology courses at the University of California, San Diego who volunteered for the experiment in order to earn course credit.

*Materials.* In the I123 condition, three plastic orange balls of different sizes (small, medium, and large) and three porcelain plates were used. In the I12-E3 condition, three plastic rings of different sizes (small, medium, and large) and three plastic poles were used. In the I1-E23 condition, three plastic cups of different sizes (small, medium, and large) and three paper plates were used. All three cups were filled with coffee. The sizes of the cups were constrained such that a larger cup could be placed on the top of a smaller cup but not vice versa, in which case the coffee would spill.

*Design.* This is a within-subject design. Each subject played all three games, one for each of the three conditions, once in a randomized order (e.g., I1-E23, I123, I12-E3). There were six possible permutations for the three games. Each permutation was assigned to a subject randomly. There were a total of eighteen subjects. Due to a limitation in the number of subjects available, the starting and ending positions were not randomized. That is, for each subject, the first, the second, and the

third games always started at positions S1, S2, and S3 and ended at positions E1, E2, and E3, respectively (see Figure 2B). The starting and ending positions should not cause significant systematic deviation because the three pairs of starting and ending positions were exactly symmetric, and the order of the three games played by each subject were randomized.

*Procedure.* Each subject was seated in front of a table and read the instructions aloud slowly. Then subjects were asked to turn the instruction sheet over and to attempt to repeat all the rules. If subjects could recite all the rules twice without error, they were instructed to start the games. Otherwise they reread the instructions and were again tested. The cycle continued until they reached the criterion. The goals were externalized by placing diagrams of the final states in front of subjects. Subjects' hand movements and speech were monitored and recorded with a video camera. The solution time, which was from when the experimenter said "start" to when a subject finished the last move, were recorded by a timer synchronized with the video camera.

## Results

The average solution times, solution steps, and errors are shown in Table 1. The *p* values for the main effects and multiple comparisons are shown in Table 2. Problem difficulty measured in solution times, solution steps, and errors for the three problems was consistent. Problem difficulty was inversely proportional to the number of external rules used. The order of difficulty was, from hardest to easiest: I123 > I12-E3 ≥ I1-E23. The difference between I12-E3 and I1-E23 was not statistically significant. All errors made were for internal rules; no errors made were for external rules (Table 3).

TABLE 1. The Results of Experiment 1

Measurements	Conditions		
	I123	I12-E3	I1-E23
Times (sec)	131.0	83.0	53.9
Steps	19.7	14.0	11.4
Errors	1.4	0.61	0.22

TABLE 2. The *p* Values of Experiment 1

Comparisons	Measurements		
	Times	Steps	Errors
Main Effect	< .05	= .05	< .005
I123 vs. I12-E3	< .1	< .1	< .03
I123 vs. I1-E23	< .01	< .02	= .001
I12-E3 vs. I1-E23	> .3	> .4	> .2

NOTE: Fisher PLSD test was used for the multiple comparisons.

TABLE 3. The Pattern of Errors in Experiment 1

Rules	Conditions		
	I123	I12-E3	I1-E23
Rule 1	1	0	4
Rule 2	10	11	0
Rule 3	14	0	0

## Discussion

Two of the three conditions in this experiment, Waitress and Oranges (I123) and Waitress and Donuts (I12-E3), were modifications of the Dish-move and Peg-move problems used by Kotovsky, Hayes, and Simon (1985), respectively. The results from the present study are consistent with their results: Subjects took more time to solve I123 than I12-E3. Kotovsky et al. only reported solution times in their study. The number of steps and errors in this study are all consistent with solution times. In this experiment, the more rules externalized, the easier the task. External representations are not just memory aids as claimed by Kotovsky et al. They have properties that are different from those of internal representations. The

nature of external representations is discussed in the General Discussion section below, but one point worth mentioning here is that subjects made no errors for external rules. Rules, once externalized, seem to be error-proof.

## EXPERIMENT 2A

Experiment 1 examined the effects of the distributions of representations between an internal and an external module on problem-solving behavior. Another factor affecting performance is the structure of problem space. Different number of rules gives rise to different problem space. Figure 3 shows that the problem space structure changes with the number of rules. How does the structural change of a problem change the difficulty of the problem and the behavior of a problem

solver? There are at least two rival factors involved. On the one hand, the fewer rules, the more paths there are from an initial state to a final state. Hence, fewer rules might make the problem easier. On the other hand, the more rules, the fewer the choices. The problem solver can simply follow where the highly constrained structure forces one to go. So, more rules might make the problem easier. My hypothesis is that the hardest problem is neither the one with the most nor the fewest rules, but one with an intermediate number of rules. Experiments 2A and 2B test this hypothesis, with Experiment 2A focusing on a change of internal rules and Experiment 2B on a change of external rules.

Experiment 2A has four conditions, with four restaurant stories similar to those in Experiment 1. All rules were internal. Condition I1 has Rule 1, Condition I13 has Rules 1 and 3, Condition I123 has Rules 1, 2, and 3, and Condition I1234 has Rules 1, 2, 3, and 4. The instructions for Condition I1234 are shown in Figure 6. The instructions for Conditions I123, I13, and I1 were exactly the same as for I1234, except that Rule 4 was absent in I123, Rules 4 and 2 absent in I13, and Rules 4, 3, and 2 absent in I1.

### Waitress and Oranges

A strange, exotic restaurant requires everything to be done in a special manner. Here is an example. Three customers sitting at the counter each ordered an orange. The customer on the left ordered a large orange. The customer in the middle ordered a medium sized orange. And the customer on the right ordered a small orange. The waitress brought all three oranges in one plate and placed them all in front of the middle customer. Because of the exotic style of this restaurant, the waitress had to move the oranges to the proper customers following a strange ritual. No orange was allowed to touch the surface of the table. The waitress had to use only one hand to rearrange these three oranges so that each orange would be placed in the correct plate, following these rules:

- **Only one orange can be transferred at a time.** (Rule 1)
- **Only the largest orange in a plate can be transferred to another plate.** (Rule 3)
- **An orange can only be transferred to a plate in which it will be the largest.** (Rule 2)
- **The small orange and the large orange can NOT be in a single plate the medium sized orange is also in that plate.** (Rule 4)

How would the waitress do this? That is, you solve the problem and show the movements of oranges the waitress has to do so that each customer will get his own orange.

FIGURE 6. The instructions for Condition I1234 of Experiment 2A. The instructions for I123, I13, and I1 were exactly the same as for I1234, except that Rule 4 was absent in I123, Rules 2 and 4 absent in I13, and Rules 2, 3, and 4 absent in I1.

## Method

*Subjects.* The subjects were 24 undergraduate students enrolled in introductory psychology courses at the University of California, San Diego, who volunteered for the experiment to earn course credit.

*Materials.* Exactly the same materials used in the Waitress and Oranges problem in Experiment 1 were used in all four conditions of the present experiment.

*Design.* Each subject played all four games, once each. There were 24 possible permutations for the four games. The 24 subjects were assigned to these permutations randomly. Due to a limitation in the number of subjects available, the first, the second, the third, and the fourth games always started at positions S1, S2, S3, and S1 and ended at positions E1, E2, E3, and E1, respectively (see Figure 2B). This treatment should not cause significant systematic deviation because the task structures of the four problems each subject solved were different from each other, and the games were randomized.

*Procedure.* The procedure was the same as in

TABLE 4. THE RESULTS OF EXPERIMENTS 2A AND 2B

Conditions	Rules	Min. Steps	Times/Min. Step	Steps/Min. Step	Errors/Min. Step
Experiment 2A					
I1	1	2	2.3	1.0	0
I13	2	4	8.8	1.9	0.06
I123	3	7	21.5	2.7	0.28
I1234	4	8	18.5	1.8	0.26
Experiment 2B					
I1	1	2	2.5	1.0	0
I1-E3	2	4	6.9	1.3	0
I1-E23	3	7	9.0	1.8	0
I1-E234	4	8	12.5	1.9	0

Experiment 1.

### Results

The results are shown in the top half of Table 4. The minimum number of steps from the starting state to the final state is 2, 4, 7, and 8 for Conditions I1, I13, I123, and I1234, respectively. In order to make meaningful comparisons, the solution times, solution steps, and errors for each condition were normalized by being divided by the number of minimum steps from the starting state to the

final state. The *p* values of the main effects and multiple comparisons are shown in the top half of Table 5. When solution times or errors are used as the difficulty measurement, the difficulty order was, from easiest to hardest:  $I1 < I13 < I1234 \leq I123$ . The difference between I1234 and I123 was not statistically significant. When solution steps are used as the difficulty measurement, the difficulty order remained the same ( $I1 < I13 \leq I1234 < I123$ ), but in this case the difference between I13 and I1234 was not statistically significant.

TABLE 5. THE *p* VALUES OF EXPERIMENTS 2A AND 2B

Comparisons	Times	Steps	Errors
Experiment 2A			
Main Effect	< .0001	< .0001	= .0001
I1 vs. I13	< .05	< .005	< .06
I1 vs. I123	< .00001	< .00001	< .001
I1 vs. I1234	< .00001	< .01	< .001
I13 vs. I123	< .02	< .01	< .005
I13 vs. I1234	< .005	> .6	< .01
I123 vs. I1234	> .36	< .003	> .75
Experiment 2B			
Main Effect	< .0001	< .0001	—
I1 vs. I1-E3	< .01	< .1	—
I1 vs. I1-E23	< .0001	< .0001	—
I1 vs. I1-E234	< .00001	< .00001	—
I1-E3 vs. I1-E23	> .18	< .01	—
I1-E3 vs. I1-E234	< .0001	< .0001	—
I1-E23 vs. I1-E234	< .03	> .4	—

NOTE. Fisher PLSD test was used for the multiple comparisons.

## EXPERIMENT 2B

Experiment 2B was exactly the same as Experiment 2A, except that Rules 2, 3, and 4 were external rather than internal. There were also four conditions in this experiment. Condition I1 (Waitress and Oranges, Figure 7A) had only Rule 1 (internal). It was exactly the same as I1 in Experiment 2A. Condition I1-E3 (Waitress and Straws, Figure 7B) had Rule 1 (internal) and Rule 3 (external). The sizes of the three straws were such that a smaller straw inside a larger one could not be moved out without the larger straw being moved away first (Rule 3). Condition I1-E23 (Waitress and Coffee, Figure 7C) had Rule 1 (internal) and Rules 2 and 3 (both external). It was actually identical to the Waitress and Coffee problem in Experiment 1. Condition I1-E234 (Waitress and Tea, Figure 7D) had Rule 1 (internal) and Rules 2, 3, and 4 (all external). The sizes of the three cups were such that a smaller cup could not be placed on the top of a larger one (Rule 2), only the topmost cup could be moved (Rule 3), and the largest cup and the smallest cup could not be placed on the top of each other (Rule 4). The instructions for Condition I1 are shown in Figure 8. The instructions for Conditions I1-E3, I1-E23, and I1-E234 were exactly the same as for Condition I1, except that the word *orange* in the instructions for Condition

I1 was replaced by the words *straw*, *cup of coffee*, and *cup of tea*, respectively.

## Method

**Subjects.** The subjects were 24 undergraduate students enrolled in introductory psychology courses at the University of California, San Diego, who volunteered for the experiment to earn course credit. The data of one subject were discarded because the videotape of Condition I1 was accidentally erased by the experimenter. (His data of I1-E3, I1-E23, and I1-E234 were in the same directions as the means.)

**Materials.** Materials for Condition I1 were the same as for the Waitress and Oranges condition in Experiment 1. For Condition I1-E3, the straws and tiny plates were made from paperboard. Materials for Condition I1-E23 were the same as for the Waitress and Coffee (I1-E23) condition in Experiment 1. For Condition I1-E234, the four cups were made from metal cans.

**Design and Procedure.** The design and procedure were the same as in Experiment 2A.

## Results

The results are shown in the lower half of Table 4. The solution times, solution steps,

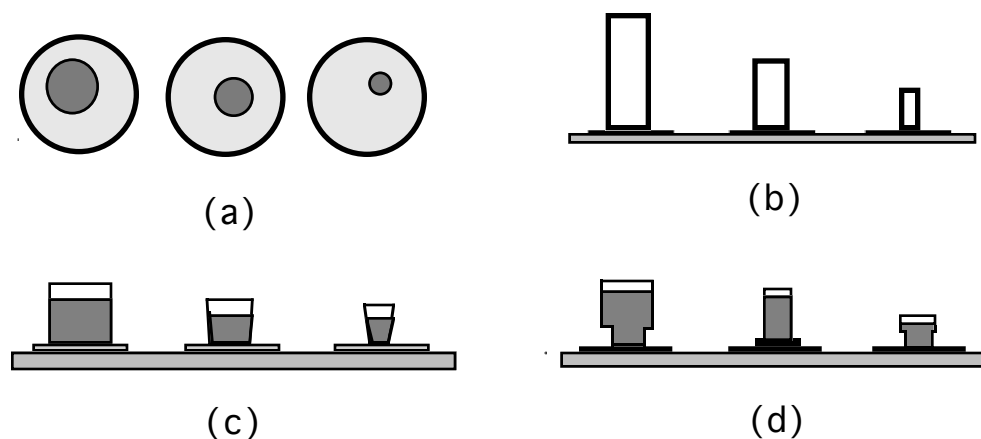


FIGURE 7. The materials used in Experiment 2B. (a), (b), (c), and (d) are for Conditions I1, I1-E3, I1-E23, and I1-E234, respectively.

and errors for all four conditions were normalized by being divided by the minimum number of steps necessary to solve the problem in each condition, as in Experiment 2A. The  $p$  values of the main effects and multiple comparisons are shown in the lower half of Table 5. If solution times are used as the difficulty measurement, the difficulty order was, from easiest to hardest:  $I1 < I1-E3 \leq I1-E23 < I1-E234$ . The difference between  $I1-E3$  and  $I1-E23$  is not statistically significant. If solution steps are used as the difficulty measurement, the difficulty order remained the same, but the difference between  $I1-E23$  and  $I1-E234$  is not statistically significant. Subjects didn't make any errors in this experiment.

## Discussion

When the rules were internal (Experiment 2A), the hardest problem was neither the one with the fewest rules ( $I1$ ), nor the one with the most rules ( $I1234$ ), but the one with an intermediate number of rules ( $I123$ ). This is what the hypothesis predicted. When all rules but one were external (Experiment 2B), however, the hardest problem was the one with the most rules ( $I1-E234$ ). Why does the nature of the rules have such an effect on the order of difficulty?

Two factors, working memory and prob-

lem structure, are involved in the determination of the order of difficulty. In Experiment 2A, all rules were internal. The working memory load was high for both  $I123$  and  $I1234$  (three and four internal rules, respectively). One possible explanation is that subjects could not do much planning because most of the working memory was used by the processing of internal rules. In this case, the structure of the problem space became the major factor determining problem difficulty. Moves were to a large extent guided by the structure of the problem space. Subjects took more steps to solve the  $I123$  problem than to solve the  $I1234$  problem, because  $I123$  was less constrained than  $I1234$ . The solution time in  $I123$  was higher than that for  $I1234$ , but not significantly, because there was one more rule in  $I1234$  and hence the time spent on this rule increased the total solution time.

In Experiment 2B, all rules but Rule 1 were external. There was little load on working memory and hence, planning was probably the dominant factor determining problem difficulty. This explains why subjects took significantly less time and fewer steps (though not significant) to solve the problems in  $I1-E23$  than in  $I1-E234$ , even though  $I1-E234$  was more constrained than  $I1-E23$ .

### Waitress and Oranges

A strange, exotic restaurant requires everything to be done in a special manner. Here is an example. Three customers sitting at the counter each ordered an orange. The customer on the left ordered a large orange. The customer in the middle ordered a medium sized orange. And the customer on the right ordered a small orange. The waitress brought all three oranges on one plate and placed them all in front of the middle customer. Because of the exotic style of this restaurant, the waitress had to move the three oranges to the proper customers following a strange ritual. No orange was allowed to touch the surface of the table. The waitress had to use only one hand to rearrange these three oranges so that each orange would be placed on the correct plate, following this rule:

- **Only one orange can be transferred at a time.** (Rule 1)

How would the waitress do this? That is, you solve the problem and show the movements of oranges the waitress has to do so that each customer will get his own orange.

FIGURE 8. This is the instruction for Condition  $I1$  of Experiment 2B. The instructions for Conditions  $I1-E3$ ,  $I1-E23$ , and  $I1-E234$  were exactly the same as this one, except that the word "orange" was replaced by "straw", "cup of coffee", and "cup of tea", respectively.

### EXPERIMENT 3

In Experiment 1, I studied the effect of the distribution of representations across internal and external modules on problem-solving behavior. In Experiments 2A and 2B, the focus was on how the nature and number of rules affected problem-solving behavior. In Experiment 3, I focus on how the distribution of representations across two internal modules (one internal module for each individual) affects group problem-solving behavior. The problem is the Waitress and Oranges version of the Tower of Hanoi with three internal rules distributed among two internal modules in three different ways. But the critical point is that the tasks are to be solved by two subjects jointly, where each subject can have different subsets of the rules.

Three restaurant stories similar to those used in Experiments 1 and 2 were made up for the three conditions. In Condition I123-I1, Subject 1 was given Rules 1, 2, and 3, and Subject 2 was given Rule 1 only. In Condition I12-I13, Subject 1 was given Rules 1 and 2, and Subject 2 given Rules 1 and 3. In Condition I123-I123, both subjects were given all the rules (1, 2, and 3).

#### Method

*Subjects.* The subjects were 42 undergraduate students at the University of California, San Diego, who were paid for participating in this experiment.

*Materials.* The materials used in this experiment were the same as those used in the Waitress and Oranges condition (I123) in Experiment 1.

*Design.* Two subjects were used in each experimental session. Each subject played the role of one of the waitresses in the restaurant stories (Kathy and Mary). Which subject played Kathy or Mary was decided randomly. In condition I123-I1, Kathy was given

Rules 1, 2, and 3, and Mary was given Rule 1 only. In I12-I13, Kathy was given Rules 1 and 3, and Mary given Rules 1 and 2. In I123-I123, Kathy and Mary were both given all three rules. The instructions for Kathy in I123-I1 are shown in Figure 9. The instructions for Mary in I123-I1 were the same as for Kathy, except that only Rule 1 was given to Mary and the names *Kathy* and *Mary* were switched. The instructions for Kathy and Mary in I12-I13 were the same as for Kathy and Mary in I123-I1, except that the rules for Kathy were Rules 1 and 3 and the rules for Mary were Rules 1 and 2. The instructions for Kathy and Mary in I123-I123 were the same as for Kathy and Mary in I123-I1, except that the rules for both Kathy and Mary were Rules 1, 2, and 3.

A between-subject design was used in this experiment. The three conditions were assigned to three pairs of subjects randomly, which constituted one block. There were seven blocks and a total of 21 pairs of subjects. Each pair of subjects played one of the three games only once. The two subjects in a pair took turns to make moves. Who made the first move was decided randomly.

*Procedure.* The two subjects in an experimental session were given three minutes to read their instructions in two separated sound-proof booths. After they finished reading their instructions, they were individually asked to recite the rules. Which subject was chosen first to recite rules was decided randomly. If a subject couldn't recite the rules the instructions were read again until all the rules could be recited without any errors. The experimenter then showed the subject two examples of each rule with the real objects that would be used in the experiment. While one subject was being tested by the experimenter, the other subject was instructed to study the rules. After the two subjects understood the game and memorized their rules, they were brought to an experimental room, there they sat side-by-side in front of a table. Once the

## Waitresses and Oranges

A strange, exotic restaurant requires everything to be done in a special manner. Here is an example. Three customers sitting at the counter each ordered an orange. The customer on the left ordered a large orange. The customer in the middle ordered a medium sized orange. And the customer on the right ordered a small orange. Two waitresses Kathy and Mary, brought all three oranges in one plate and placed them all in front of the middle customer. Because of the exotic style of this restaurant, Kathy and Mary had to move the oranges to the proper customers following a strange ritual. No orange was allowed to touch the surface of the table.

Kathy and Mary moved the oranges in turn. Because they may have different rules in mind, they have to absolutely respect each other's rules. If Kathy makes a move which is not consistent with Mary's rules, Mary should say "NO" and Kathy should try another move until it is consistent with the rules of both of them. Mary has to respect Kathy's rules in the same way. No discussion was allowed between them. The only word they could say was "NO" when a move by one of them violated the rules of the other person.

Suppose you are Kathy. You have to use only one hand to rearrange these three oranges in cooperation with Mary so that each orange will be placed in the correct plate, following these rules:

- **Only one orange can be transferred at a time.** (Rule 1)
- **Only the largest orange in a plate can be transferred to another plate.** (Rule 3)
- **An orange can only be transferred to a plate in which it will be the largest.** (Rule 2)

How would you do this? That is, you and Mary solve the problem and show the movements of oranges you have to make so that each customer will get his own orange.

FIGURE 9. The instructions for Kathy in *I123-I1*.

experimenter said "go," they started to play the game, making moves in turn. They were forbidden to communicate with each other by any means (verbal or nonverbal) throughout the experiment, except that they could say the word "no" when one person made a move that was not consistent with the rules of the other person. The game-playing sessions were video-recorded.

### Results

The average solution times, solution steps, and errors are shown in the columns under Experiment 3 in Table 6. The  $p$  values of the main effects and multiple comparisons are shown in Table 7. The difficulty order in both solution times and steps was, from easiest to hardest:  $I123-I123 < I123-I1 < I12-I13$ . All time differences were significant, but the difference of steps between *I12-I13* and *I123-I1* was not statistically significant. There were two kinds of errors in this experiment (Table 6). Within-subject errors were moves that violated a subject's own rules. Between-

subject errors were moves that violated the other subject's rules. The errors were not significantly different across conditions.

### Discussion

The problem was easiest when the two subjects both knew the full set of rules (*I123-I123*). The condition in which only one knew all the rules (*I123-I1*) was easier than the one in which neither knew all (*I12-I13*). When the data of Experiment 3 were compared with the between-subject data of Condition *I123* in Experiment 1 (see Table 6), an unexpected result was found: Two persons both knowing the full set of rules (*I123-I123*) could solve the problem better than could one person alone with the full set of rules (*I123*), even if the two did not communicate. To test whether the difference between conditions *I123-I123* of Experiment 3 and *I123* of Experiment 1 might be due to the difference between the instructions, a post hoc condition, *I123\**, was added to Experiment 3. In *I123\**, the instructions and procedure were the same as for

TABLE 6. THE RESULTS OF EXPERIMENT 3

	Experiment 3			Expt. 1	Posthoc
	I123-I1	I12-I13	I123-I123	I123	I123*
Times	231.1	391.0	114.9	215.2	215.5
Steps	21.3	26.3	13.6	26.2	24.9
Within-Subject Errors	1.6	3.0	1.3	2.0	3.9
Between-Subject Errors	10.1	10.4	—	—	—

NOTE. I123\* is a supplementary experiment that was done after Experiment 3 had been completed. The data of I123 were from the first game each subject played in Experiment 1, because Experiment 3 was a between-subject design. See text in the discussion section for detail.

I123-I123, except that just before the experiment, the subject was told to play the game alone instead of with another subject. The results are shown in Tables 6 and 7. It supports the observation that two people were better than one at solving the problem.

## GENERAL DISCUSSION

A task can be represented in a distributed cognitive system as a set of modules, some internal and some external. The difficulty of a problem and the behavior of the problem solver are determined by many factors, including the structure of the abstract task space and the distribution of representations within the set of modules involved in the task. Given the initial and final states of a task, the structure of the abstract task space depends upon the number of rules. Experiments 2A and 2B showed that the difficulty of a task did not increase monotonically with the number of rules in all cases. When all rules were internal, the most difficult problem was the one with an intermediate number of rules (three), not the one with the most (four). The

behavior of the problem solvers differed because the structures of the problem spaces differed.

Experiments 1 and 3 showed that given the same abstract task space, different distributions of the representations among the internal and external modules caused different problem difficulties and problem-solving behavior. Is a problem always easier when the representations are distributed among several modules than when the representations are concentrated in one module? Experiment 1 showed that when a problem was represented across an internal module and an external module, shifting the representations from the internal module to the external module made the problem easier (Condition I1-E23 was easier than I123). When the problem was represented across two internal modules (Experiment 3), however, shifting representations from one internal module to another only made the problem more difficult (Condition I12-I13 was more difficult than I123-I1). This implies that simply distributing representations across modules does not necessarily improve performance. How can one explain these

TABLE 7. THE *p* VALUES OF EXPERIMENT 3

Comparisons	Times	Steps	Within-Subject Errors	Between-Subject Errors
Main Effect	< .005	< .02	> .3	> .9
I123- vs. I12-I13	< .05	> .24	—	> .9
I123-I1 vs. I123-I123	= .1	= .06	—	—
I12-I13 vs. I123-I123	< .005	< .006	—	—
I123* vs. I123-I123	= .1	< .1	> .18	—

NOTE. Fisher PLSD test was used for the multiple comparisons.

results? I propose a “shareability” hypothesis, but before elaborating this hypothesis, I first discuss the nature of external representations.

### The Nature of External Representations

External objects are not just peripheral aids to cognition, they provide a different form of representation—an external representation. External representations have four properties:

1. They are external memory aids;
2. They do not need for interpretation;
3. They structure cognitive behavior;
4. They change the task.

1. *External representations provide external memory aids.* For example, for all of the games in the present study, the goal problem states didn’t need to be remembered, because they were represented by the pictures placed in front of the subjects.

2. *External representations can provide information that can be directly perceived and used without being interpreted and formulated explicitly.* In contrast, internal rules have to be interpreted or transformed in order to be utilized. For example, in the Waitress and Coffee (I1-E23) version of the Tower of Hanoi, Rules 2 and 3 were not told to the subjects: They were built into the physical constraints and perceived and followed directly. When the subjects were asked to formulate the rules after playing the game, few could do it. (This property has been extensively discussed by Gibson, 1979, in the theory of affordances and Norman, 1988, in the theory of external knowledge.)

3. *External representations structure cognitive behavior.* The physical structures in the external world constrain the range of possible cognitive behaviors in the sense that some behaviors are allowed and others prohibited. For example, in the case of the Waitress and Coffee (I1-E23) version of the Tower of

Hanoi, external Rules 2 and 3 construct the external problem space, which is part of the structure of the cognitive task. In this sense, external Rules 2 and 3 structure the cognitive behavior.

4. *External representations change the nature of the task.* Norman (1990) proposed that external representations change the nature of the task from the task performer’s point of view and enhance the system’s ability from the system’s (task performer + external representation) point of view. Consider an example. In the I123 condition, a problem solver had to process three internal rules, whereas in the I1-E23 condition, the problem solver only had to process one internal rule. The cognitive processes of the task performers were different in these two conditions. Nevertheless, the performance of the system I1-E23 was much better than the system I123.

External representations do much more than simply serve as a memory aid. Distributed cognitive activities are produced by the interaction of internal and external representations.

### The Shareability Hypothesis

I propose the following “shareability” hypothesis: In a multimodular distributed cognitive system, with all else consistent, the more knowledge shared among the modules, the better the system’s performance.

There are two basic kinds of shared knowledge. The first is the knowledge shared between internal modules (in the present study, one internal module for each individual). The second is the knowledge shared between an internal and an external module. In Experiment 3, the shared knowledge between the two internal modules of I123-I1 is Rule 1, that between the two internal modules of I12-I13 is also Rule 1, and that between the two internal modules of I123-I123 is Rules 1, 2 and 3. Because the shared knowledge in I123-I123 is more than that in I123-I1 and that in I12-I13, the

performance of the system I123-I123 is better than those of systems I123-I1 and I12-I13.

When the knowledge of an external module can be perceived directly by an internal module (Property 2 of external representations), we can say that the knowledge of the external module is possessed by the internal module "on-line." In Experiment 1, the shared knowledge between the internal and external modules of I1-E23 is Rules 2 and 3. Here, the knowledge in the external module is shared on-line by the internal module because Rules 2 and 3 in the external module can be perceived and followed directly by the internal module. By the same argument, there is no shared knowledge between the internal and the external modules of I123 (I123-E0), while E3 is shared in I12-E3. Based on the shareability hypothesis, the performance of system I1-E23 should be better than I12-E3, which in turn should be better than I123. This is what the results of Experiment 1 showed.

## CONCLUSION

Modularity of representations is a useful framework for the analysis of distributed cognitive activities. Under this framework, distributed cognitive activities are produced by the interaction of internal and external representations. External representations are not peripheral aids, they are an indispensable part of cognition. One of the factors determining the performance of a distributed cognitive system is the sharing of knowledge among modules—between internal and internal modules of different individuals and between internal and external modules.

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